

# X100/301

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NATIONAL  
QUALIFICATIONS  
2008

TUESDAY, 20 MAY  
9.00 AM – 10.30 AM

MATHEMATICS  
HIGHER  
Paper 1  
(Non-calculator)

**Read carefully**

**Calculators may NOT be used in this paper.**

**Section A – Questions 1–20 (40 marks)**

Instructions for completion of **Section A** are given on page two.  
For this section of the examination you must use an **HB pencil**.

**Section B (30 marks)**

- 1 Full credit will be given only where the solution contains appropriate working.
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## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

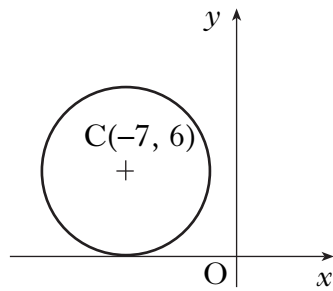
1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0.3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of  $u_{12}$ ?

- A 6.6
- B 7.8
- C 8.7
- D 9.6

2. The  $x$ -axis is a tangent to a circle with centre  $(-7, 6)$  as shown in the diagram.



What is the equation of the circle?

- A  $(x + 7)^2 + (y - 6)^2 = 1$
- B  $(x + 7)^2 + (y - 6)^2 = 49$
- C  $(x - 7)^2 + (y + 6)^2 = 36$
- D  $(x + 7)^2 + (y - 6)^2 = 36$

3. The vectors  $\mathbf{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$  are perpendicular.

What is the value of  $k$ ?

- A 0
- B 3
- C 4
- D 5

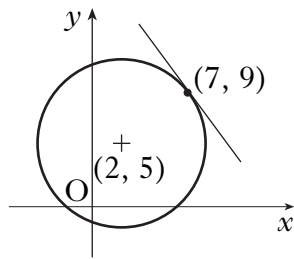
4. A sequence is generated by the recurrence relation  $u_{n+1} = 0.4u_n - 240$ .

What is the limit of this sequence as  $n \rightarrow \infty$ ?

- A -800
- B -400
- C 200
- D 400

5. The diagram shows a circle, centre  $(2, 5)$  and a tangent drawn at the point  $(7, 9)$ .

What is the equation of this tangent?



A  $y - 9 = -\frac{5}{4}(x - 7)$

B  $y + 9 = -\frac{4}{5}(x + 7)$

C  $y - 7 = \frac{4}{5}(x - 9)$

D  $y + 9 = \frac{5}{4}(x + 7)$

**[Turn over**

6. What is the solution of the equation  $2 \sin x - \sqrt{3} = 0$  where  $\frac{\pi}{2} \leq x \leq \pi$ ?

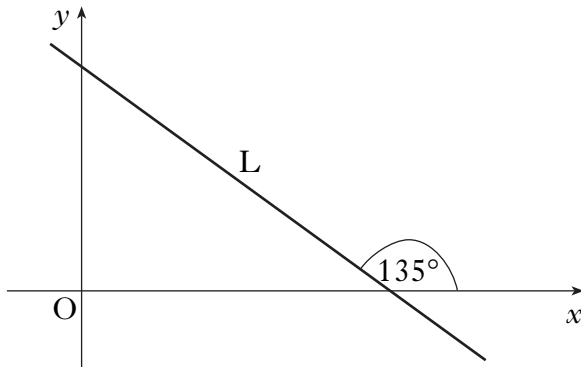
A  $\frac{\pi}{6}$

B  $\frac{2\pi}{3}$

C  $\frac{3\pi}{4}$

D  $\frac{5\pi}{6}$

7. The diagram shows a line L; the angle between L and the positive direction of the  $x$ -axis is  $135^\circ$ , as shown.



What is the gradient of line L?

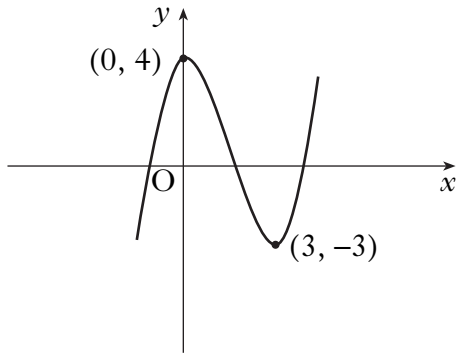
A  $-\frac{1}{2}$

B  $-\frac{\sqrt{3}}{2}$

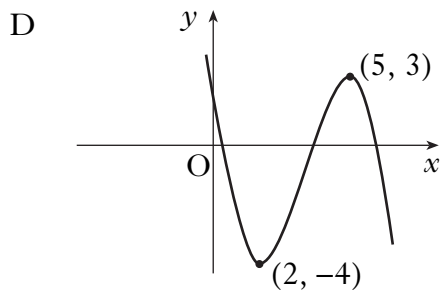
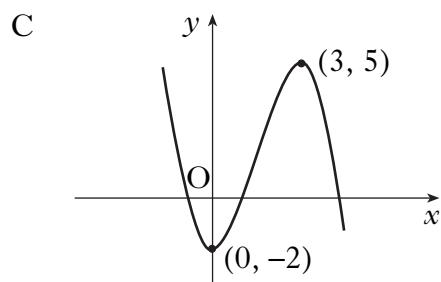
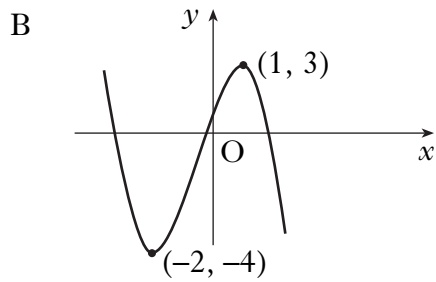
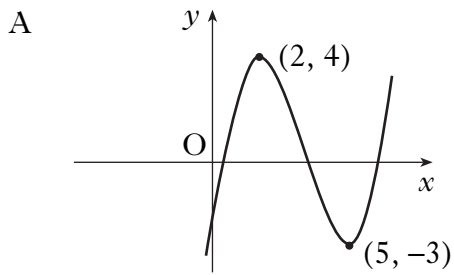
C  $-1$

D  $\frac{1}{2}$

8. The diagram shows part of the graph of a function with equation  $y = f(x)$ .



Which of the following diagrams shows the graph with equation  $y = -f(x - 2)$ ?



9. Given that  $0 \leq a \leq \frac{\pi}{2}$  and  $\sin a = \frac{3}{5}$ , find an expression for  $\sin(x + a)$ .

A  $\sin x + \frac{3}{5}$

B  $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C  $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D  $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

10. Here are two statements about the roots of the equation  $x^2 + x + 1 = 0$ :

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. E(-2, -1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

What is the ratio in which P divides EF?

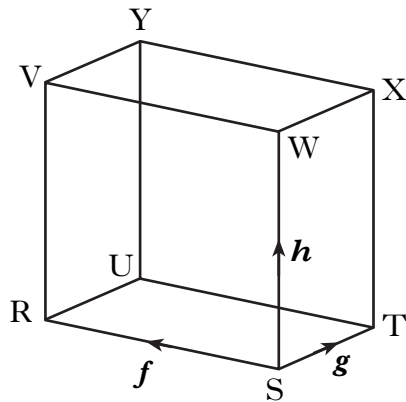
A 1:1

B 1:2

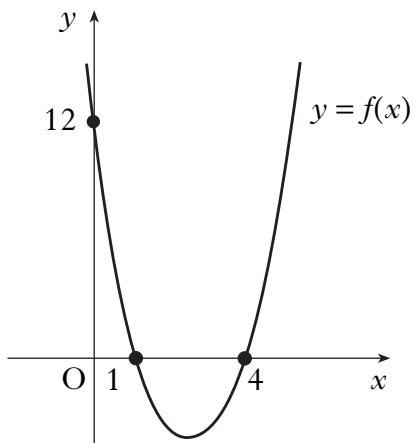
C 1:4

D 1:6

12. In the diagram RSTU, VWXY represents a cuboid.  
 $\vec{SR}$  represents vector  $f$ ,  $\vec{ST}$  represents vector  $g$  and  $\vec{SW}$  represents vector  $h$ .  
 Express  $\vec{VT}$  in terms of  $f$ ,  $g$  and  $h$ .



- A  $\vec{VT} = f + g + h$   
 B  $\vec{VT} = f - g + h$   
 C  $\vec{VT} = -f + g - h$   
 D  $\vec{VT} = -f - g + h$
13. The diagram shows part of the graph of a quadratic function  $y = f(x)$ .  
 The graph has an equation of the form  $y = k(x - a)(x - b)$ .



What is the equation of the graph?

- A  $y = 3(x - 1)(x - 4)$   
 B  $y = 3(x + 1)(x + 4)$   
 C  $y = 12(x - 1)(x - 4)$   
 D  $y = 12(x + 1)(x + 4)$



14. Find  $\int 4 \sin(2x + 3) dx$ .
- A  $-4 \cos(2x + 3) + c$
  - B  $-2 \cos(2x + 3) + c$
  - C  $4 \cos(2x + 3) + c$
  - D  $8 \cos(2x + 3) + c$
15. What is the derivative of  $(x^3 + 4)^2$ ?
- A  $(3x^2 + 4)^2$
  - B  $\frac{1}{3}(x^3 + 4)^3$
  - C  $6x^2(x^3 + 4)$
  - D  $2(3x^2 + 4)^{-1}$
16.  $2x^2 + 4x + 7$  is expressed in the form  $2(x + p)^2 + q$ .  
What is the value of  $q$ ?
- A 5
  - B 7
  - C 9
  - D 11
17. A function  $f$  is given by  $f(x) = \sqrt{9 - x^2}$ .  
What is a suitable domain of  $f$ ?
- A  $x \geq 3$
  - B  $x \leq 3$
  - C  $-3 \leq x \leq 3$
  - D  $-9 \leq x \leq 9$

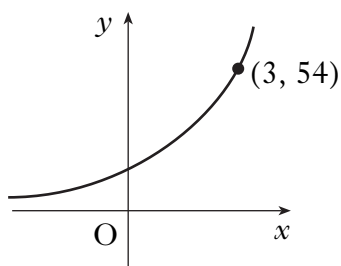
18. Vectors  $\mathbf{p}$  and  $\mathbf{q}$  are such that  $|\mathbf{p}| = 3$ ,  $|\mathbf{q}| = 4$  and  $\mathbf{p} \cdot \mathbf{q} = 10$ .

Find the value of  $\mathbf{q} \cdot (\mathbf{p} + \mathbf{q})$ .

- A 0
- B 14
- C 26
- D 28

19. The diagram shows part of the graph whose equation is of the form  $y = 2m^x$ .

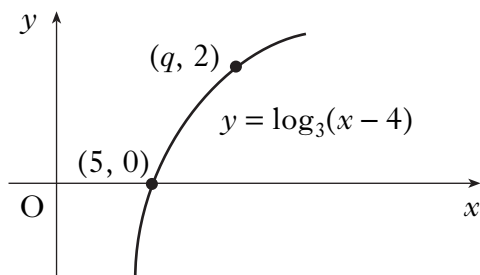
What is the value of  $m$ ?



- A 2
- B 3
- C 8
- D 18

20. The diagram shows part of the graph of  $y = \log_3(x - 4)$ .

The point  $(q, 2)$  lies on the graph.



What is the value of  $q$ ?

- A 6
- B 7
- C 8
- D 13

[END OF SECTION A]

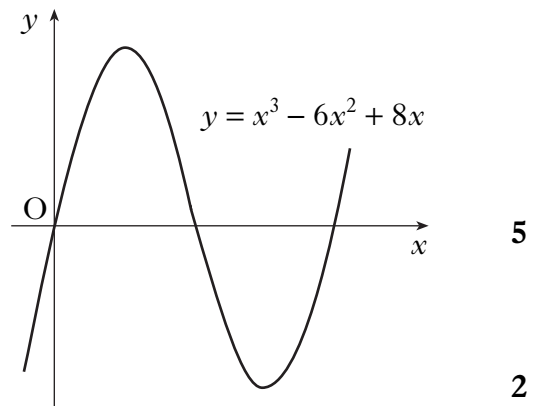
**SECTION B**

**ALL questions should be attempted.**

*Marks*

21. A function  $f$  is defined on the set of real numbers by  $f(x) = x^3 - 3x + 2$ .
- (a) Find the coordinates of the stationary points on the curve  $y = f(x)$  and determine their nature. 6
- (b) (i) Show that  $(x - 1)$  is a factor of  $x^3 - 3x + 2$ .  
(ii) Hence or otherwise factorise  $x^3 - 3x + 2$  fully. 5
- (c) State the coordinates of the points where the curve with equation  $y = f(x)$  meets both the axes and hence sketch the curve. 4

22. The diagram shows a sketch of the curve with equation  $y = x^3 - 6x^2 + 8x$ .



- (a) Find the coordinates of the points on the curve where the gradient of the tangent is  $-1$ .
- (b) The line  $y = 4 - x$  is a tangent to this curve at a point A. Find the coordinates of A.

23. Functions  $f$ ,  $g$  and  $h$  are defined on suitable domains by

$$f(x) = x^2 - x + 10, g(x) = 5 - x \text{ and } h(x) = \log_2 x.$$

- (a) Find expressions for  $h(f(x))$  and  $h(g(x))$ . 3
- (b) Hence solve  $h(f(x)) - h(g(x)) = 3$ . 5

[END OF SECTION B]

[END OF QUESTION PAPER]

# **X100/302**

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2008

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Paper 2

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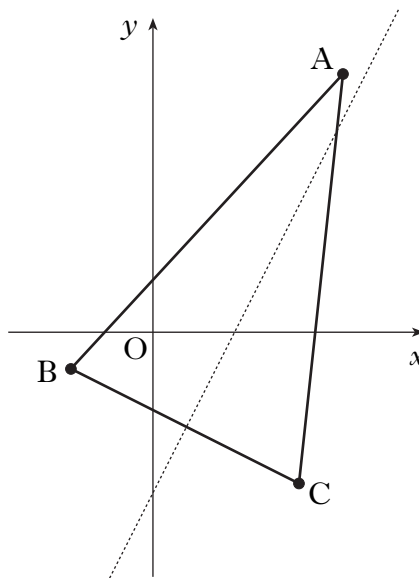
ALL questions should be attempted.

Marks

1. The vertices of triangle ABC are A(7, 9), B(-3, -1) and C(5, -5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

- (a) Show that the equation of the perpendicular bisector of BC is  $y = 2x - 5$ .
- (b) Find the equation of the median from C.
- (c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.



4

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3

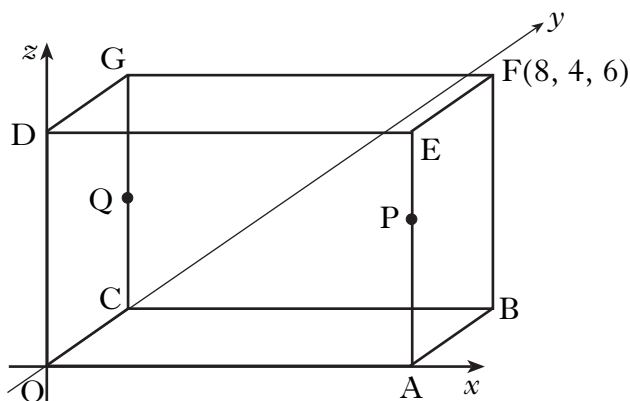
2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.

- (a) State the coordinates of P and Q.
- (b) Write down the components of  $\vec{PQ}$  and  $\vec{PA}$ .
- (c) Find the size of angle QPA.



2

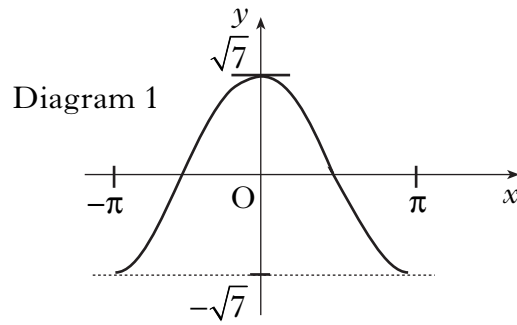
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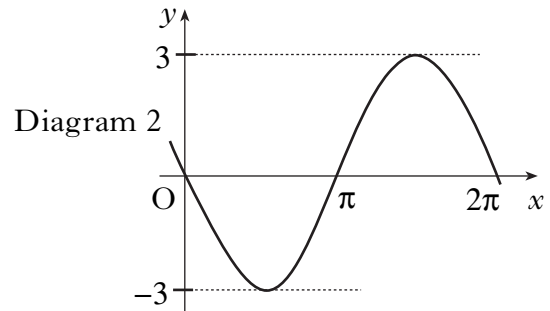
3. (a) (i) Diagram 1 shows part of the graph of  $y = f(x)$ , where  $f(x) = p \cos x$ .

Write down the value of  $p$ .



- (ii) Diagram 2 shows part of the graph of  $y = g(x)$ , where  $g(x) = q \sin x$ .

Write down the value of  $q$ .



- (b) Write  $f(x) + g(x)$  in the form  $k \cos(x + a)$  where  $k > 0$  and  $0 < a < \frac{\pi}{2}$ . 4
- (c) Hence find  $f'(x) + g'(x)$  as a single trigonometric expression. 2

4. (a) Write down the centre and calculate the radius of the circle with equation  $x^2 + y^2 + 8x + 4y - 38 = 0$ . 2

- (b) A second circle has equation  $(x - 4)^2 + (y - 6)^2 = 26$ .

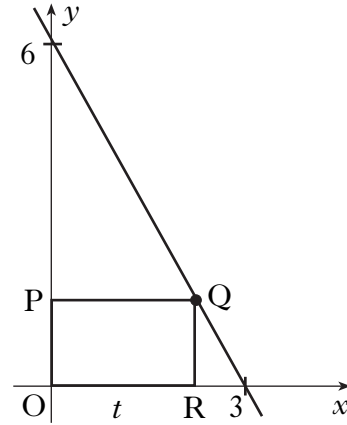
Find the distance between the centres of these two circles and hence show that the circles intersect. 4

- (c) The line with equation  $y = 4 - x$  is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. 5

5. Solve the equation  $\cos 2x^\circ + 2\sin x^\circ = \sin^2 x^\circ$  in the interval  $0 \leq x < 360$ . 5

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).  
 OPQR is a rectangle, where P and R lie on the axes and  $OR = t$ .
- (a) Show that  $QR = 6 - 2t$ .
- (b) Find the coordinates of Q for which the rectangle has a maximum area.



3

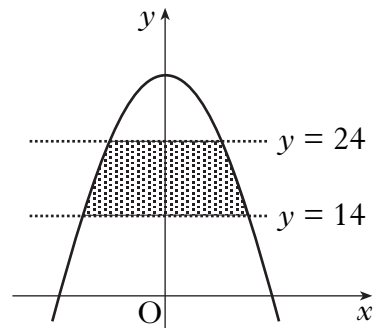
6

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines  $y = 14$  and  $y = 24$ .

Calculate the shaded area.



8

[END OF QUESTION PAPER]